

P. R. GOVERNMENT COLLEGE (A), KAKINADA
III B.Sc. MATHEMATICS - Semester V (w.e.f. 2018-2019)
Paper V: Ring Theory & Vector Calculus

Total Hrs. of Teaching Learning & Evaluation: 75 @ 6 h / Week

Total Credits: 05

OBJECTIVES:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between line Integral, Surface Integral and Volume integral.
- To introduce the concepts of geometrical meaning of Gradient, Divergence and Curl

RING THEORY

Unit – I: Rings – I

(14 Hrs)

Definition of Ring and Basic Properties, Boolean Rings, Divisors of Zero and Cancellation Laws in Rings, Integral Domain, Division Ring and Fields, The Characteristic of a Ring – The Characteristic of an Integral Domain, the Characteristic of a Field, Sub Rings and Ideals.

Unit – II: Rings – II

(12 Hrs)

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT – III: Vector Differentiation

(12 Hrs)

Vector differentiation – Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient, Divergence, Curl operators, Formulae involving these operators.

UNIT – IV: Vector Integration

(12 Hrs)

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications

(10 Hrs)

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theorems.

Co-Curricular: Assignment, Seminar, Quiz, etc.

(15 Hrs)

Additional Inputs: Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I. N. Herstine
2. Abstract Algebra by J. Fraleigh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.C hand & Company Pvt. Ltd., New Delhi

4. Vector Calculus by R. Gupta, Published by Laxmi Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-V, PAPER-V

Unit	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Rings – I	01	01	02	22
II	Rings – II	01	01	02	22
III	Vector differentiation	01	01	02	22
IV	Vector integration	01	01	01	14
V	Vector Integration Applications	01	01	01	14
TOTAL		05	05	08	94

E.Q = Essay questions (8 marks)

S.A.Q = Short answer questions (5 marks)

V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 5 x 8 M = 40

Short answer questions : 3 x 5 M = 15

Very Short answer questions : 5 x 1 M = 05

Total Marks _____ = 60

P.R. Government College (Autonomous), Kakinada
III year B.Sc., Degree Examinations - V Semester
Mathematics: Ring Theory & Vector Calculus
Paper – V (Model Paper w. e. f. 2019-2020)

Time: 2 Hrs 30 Min

Max. Marks: 60M

PART – I

Answer ALL the following questions. Each question carries 1 mark.

5 x 1 = 5 M

1. Define Boolean Ring.
2. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \rightarrow Z(\sqrt{2})$ defined by $f(m + n\sqrt{2}) = m - n\sqrt{2}, \forall m + n\sqrt{2} \in Z(\sqrt{2})$.
3. Find div f, where $f = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
4. Evaluate $\int_0^1 (e^t \bar{i} + 2e^{-2t} \bar{j}) dt$.
5. State Green's Identities.

PART –II

Answer any THREE of the following questions. Each question carries 5 marks.

3 x 5 = 15

M

6. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
7. Let R and R' be two rings and $f: R \rightarrow R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
8. Prove that $\text{div Curl } \bar{f} = 0$.
9. If $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of $\bar{F}$ round the curve C, where C is the Circle $x^2 + y^2 = 1, z = 0$.
10. Evaluate $\oint_C (\cos x \cdot \sin y - xy) dx + \sin x \cdot \cos y dy$, by Green's theorem, where C is the circle $x^2 + y^2 = 1$.

PART –III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5 X 8 = 40 M

SECTION – A

11. If $E = \{0, 1, 2, 3, 4\}$, then prove that $(E, +_5, \times_5)$ form a field. Justify your answer.
12. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
13. State and prove fundamental theorem of homomorphism of rings.
14. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

SECTION – B

15. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$.
16. Find the directional derivative of $f = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
17. If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$, evaluate $\int \vec{F} \cdot \vec{N} \, dS$ where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$.
18. Verify Stokes theorem for $A = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.
